The Vacuum.Net Theory: A Self-Contained Statement

Axioms, proved results, physical correspondences, and independent machine verification

J. Konstapel, Constable Research, Leiden — with computational assistance from Claude (Anthropic) · 29 October 2026 · Canonical sources: *The Vacuum.Net Theory: Foundations*, 9th ed., v9.2, and *The Vacuum.Net Theory: Physics*, 9th ed., v9.0 (both 27 October 2026) · Verification: maze.swarp.nl/bewezen

Abstract. The Vacuum.Net theory starts from one object, a strand that returns to places it has visited, and one rule, that only what closes persists. Its register has three values, −1, 0 and +1, which are the digits of balanced ternary. This paper states the theory in one unambiguous form. Section 2 gives the definitions and the five axioms. Sections 3–6 state the mathematical results (T1–T24 of the *Foundations*) with their proofs or proof ideas and their hypotheses. These results make no physical claim. Section 7 states the connection to physics (the *Physics* document) and keeps four kinds of statement apart: theorems that are proved under a named physical input, correspondences, identifications, and predictions. Section 8 reports an independent verification. Eighteen of the mathematical results were proved in the proof assistant Lean 4, using only its standard axioms; fourteen further results were recomputed exactly. Four rounds of independent review fixed what each result does and does not establish. Every claim in this paper carries one of five statuses: assumed, proved, conditionally proved, corresponded, or predicted. Open questions are listed as open.

1. Purpose and reading guide

Earlier statements of the theory exist in several editions. They use different numbering and differ in scope. This paper fixes one statement. It follows the ninth edition, which splits the theory into two documents: *Foundations* (mathematics only) and *Physics* (the connection to physics). Theorems are cited as "F-Tn" for the Foundations and "P-Tn" for Physics.

Each result carries a verification mark:

- `Lean` proved in Lean 4, with no unproved assumptions beyond Lean's standard axioms (propositional extensionality, choice, quotient soundness);
- `exact` recomputed exactly (integers, fractions, symbolic algebra) on the stated domain;
- `not verified` proved in the edition but not independently checked here.

A mark applies to the part of the statement named in Section 8. Where a statement says more than was checked, the excess is listed there.

2. Definitions and axioms

DEFINITIONS

An ordered index set I (the course of the strand). A set P of places. The *strand* is a map $\sigma: I \to P$. A *loop* is a pair $i < j$ with $\sigma(i) = \sigma(j)$. A *node* is a place visited at least twice. A *passage* through a node is an index i with $\sigma(i)$ at that node; the first passage is the least such index.

THE GROUND

The trinity: three values −1, 0, +1, written T, 0, 1, whose sum is zero. They are the digits of balanced ternary (F-T1).

N1 (STRAND) There is exactly one strand. Every distinction is a loop.

N2 (CLOSURE) A loop persists if and only if it lies in a closed window. What does not close passes on as tension. Closure is the selection rule; there is no external selector.

N3 (MEMORY) At every node the passages are ordered. The first passage fixes the direction; later passages are taken relative to it.

N4 (CONTEXT) The response of the medium is a state function, not a universal constant. Universality is the empirical limiting case in which the state variable does not vary across the sample.

N5 (SCALE) Merged into blocks, the coarse strand obeys N1 to N4 again. What does not close at one scale is the material of the next.

TRIT RULE

Let i be a later passage through a node, not the last index, and let f be the first passage through $\sigma(i)$. Then $t(i) = +1$ if $\sigma(i+1) = \sigma(f+1)$; $t(i) = -1$ if $\sigma(i+1) = \sigma(f-1)$; $t(i) = 0$ otherwise.

WINDOW

A window is three consecutive trits. It *closes* when the sum of its trits is 0 modulo 3.

3. The ternary register (F-T1 to F-T11)

F-T1 (THE TRINITY AS REGISTER) Lean

For a sum s of trits, let the digit $d \in \{-1, 0, +1\}$ be the balanced residue with $d \equiv s \pmod 3$, and the carry $(s - d)/3$. (i) The digit is 0 if and only if $s \equiv 0 \pmod 3$. (ii) Whatever is not kept passes on whole as carry to the next scale. (iii) The residues form $\mathbb{Z}_3$, the smallest cyclic group in which a value differs from its reverse.

Proof. Every integer has exactly one balanced residue (F-T12). In $\mathbb{Z}_1$ and $\mathbb{Z}_2$ every element equals its reverse ($1 = -1$ in $\mathbb{Z}_2$); in $\mathbb{Z}_3$ it does not. ∎

F-T2 (WINDOW OF THREE) Lean

A run of equal nonzero trits first leaves no residue at length three.

Proof. A run of $\mathbf{k}$ equal trits $\mathbf{x} \neq 0$ has sum $\mathbf{kx}$, divisible by 3 first at $\mathbf{k} = 3$. ∎

F-T3 (NINE OF TWENTY-SEVEN) Lean

Exactly 9 of the 27 windows close. They form a subgroup of $\mathbb{Z}_3^3$ of index 3.

Proof. The sum map $\mathbb{Z}_3^3 \to \mathbb{Z}_3$ is a surjective homomorphism; the closing windows are its kernel. ∎

F-T4 (TWO KINDS OF CLOSURE) Lean

A window closes if and only if its trits are all equal or all different: 3 homogeneous and 6 heterogeneous closing windows. The 18 open windows have exactly two equal trits.

Proof. All equal gives $3\mathbf{a} \equiv 0$; all different gives $1 + 0 - 1 = 0$; exactly two equal gives $2\mathbf{a} + \mathbf{b} \equiv 0$, hence $\mathbf{b} \equiv \mathbf{a}$, a contradiction. ∎

F-T5 (THE TRIT CUBE) Lean

In $\{\mathrm{T}, 0, 1\}^3$ the cells with k nonzero coordinates number 1, 6, 12, 8. The closing cells are the centre, the six cells with two nonzero coordinates of opposite sign, and (1, 1, 1) and (T, T, T): $1 + 0 + 6 + 2 = 9$.

F-T6 (CLOSURE PROBABILITY) exact

For independent trits with law (p_0, p_+, p_-) and $\hat{p}(z) = p_0 + p_+ z + p_- z^2$, a window closes with probability $(1/3)\,[1 + \hat{p}(\omega)^3 + \hat{p}(\omega^2)^3]$, $\omega = e^{2\pi i/3}$. The uniform law gives 1/3.

F-T7 (ELEMENTARY CLOSURES) Lean exact

Every closed trit word reduces to nothing by removing, one at a time, a standing 0, a meeting $(x, -x)$ or a ring (x, x, x). No open word does. Among closures with nonzero steps, a meeting is the shortest with winding number 0 and a ring the shortest with winding number ±1.

Proof. Each removal keeps the sum modulo 3, so only closed words reduce. Conversely, remove zeros; if neighbours $\mathbf{x}, -\mathbf{x}$ occur, remove them; otherwise all entries are equal, their number is divisible by 3, and the word splits into rings. Induction on length. ∎ (In Lean for words of every length.)

F-T7.1 (WHAT A CLOSURE CARRIES) Lean

A standing and a meeting close with carry 0. A ring closes with carry ±1: $1 + 1 + 1 = 10$ and $\mathrm{T} + \mathrm{T} + \mathrm{T} = \mathrm{T}0$ in balanced ternary.

F-T8 (PROPAGATION; NINE WORDS) Lean exact

With s_k the sum of window k, $s_{k+1} \equiv s_k + t_{k+3} - t_k \pmod 3$. A word of length $n \geq 3$ closes in every window if and only if its first window closes and it has period 3. For every $n \geq 3$ there are exactly 9 such words.

F-T9 (A RING THAT CLOSES EVERYWHERE) Lean exact

If every window of a ring of $n \geq 3$ trits closes, then $t_{i+3} = t_i$. With labelled positions there are 9 such rings when 3 divides n, and 3 otherwise; as rotation classes, 5 and 3.

F-T10 (THE REWRITE OF THE REGISTER) Lean

Let $R(t)_i = -(t_{i-1} + t_{i+1})$. A ring is a fixed point of R if and only if every window closes.

F-T11 (SEAM RULE) Lean

For closed windows A and B joined as $A_1A_2A_3B_1B_2B_3$, the number of open windows at the seam is $[A_1 \neq B_1] + [A_3 \neq B_3]$. The seam is closed if and only if $B = A$. Over the 81 ordered pairs: 9 closed seams, 36 with one open window, 36 with two.

4. Addresses (F-T12 to F-T14)

F-T12 (UNIQUE ADDRESS) Lean

Every nonzero integer has exactly one finite balanced-ternary representation $\Sigma\, d_k 3^k$ with $d_k \in \{T, 0, 1\}$ and nonzero leading trit. Words of k digits give exactly the integers from $-(3^k - 1)/2$ to $(3^k - 1)/2$.

Proof. Existence: shift N by $(3^k - 1)/2$ and use the standard ternary expansion. Uniqueness: a nonzero difference $\Sigma\, \mathbf{d}_i 3^i$ with $|\mathbf{d}_i| \leq 2$ has a largest nonzero term of size $\geq 3^k$, while the rest is at most $3^k - 1$. ∎

Corollaries. F-T12.1: the address of $-Q$ is the address of Q mirrored trit-wise Lean. F-T12.2: the discarded tail of a finite word with L digits is strictly smaller than half a unit, so truncation is rounding Lean. F-T12.3: the largest value with n digits is $(3^n - 1)/2$, and $n \geq 1$ has $\lfloor \log_3(2n - 1) \rfloor + 1$ digits Lean exact.

F-T13 (TWO COUNTS) exact

Of the 3^n open words of length $n \geq 3$, 3^{n-1} close their first window. Closed configurations of n trits without a starting point number $g(n) = (1/n)\, \Sigma_{d|n}\, \phi(n/d)\, 3^d = 3, 6, 11, 24, 51, 130$ for $n = 1, \ldots, 6$ (Burnside).

F-T14 (REST AND TURN IN $\mathbb{Z}[\omega]$) exact

Let $\beta = 1 - \omega$, $d(z)$ the balanced residue of $z = a + b\omega$, and $T(z) = (z - d(z))/\beta$. Every orbit of T reaches 0 or one of the fixed points $\pm(1 + \omega)$, sixth roots of unity. An element has a finite trit word if and only if its orbit reaches 0; not every element does ($2 \to 2 + \omega \to 1 + \omega$).

5. Two ladders (F-T15 to F-T18)

> F-T15 (LADDER) Lean
> *Hypothesis:* at each scale the strand merges into blocks of equal size, each one elementary closure of length 2 or 3. Then after a merges of meetings and b merges of rings, one coarse passage contains $2^a \cdot 3^b$ finest passages.

> F-T16 (NO EXACT MEETING) Lean
> $2^n \neq 3^k$ for all integers n, k other than $n = k = 0$. So $\log_2 3$ is irrational.

Proof. For $\mathbf{n} > 0$, 2^n is even and 3^k odd; for $\mathbf{n} = 0$, $3^k = 1$ forces $\mathbf{k} = 0$. Negative exponents reduce to these cases. ∎

> F-T17 (THE CLOSURE DEPTHS) exact
> A closure depth is a $k \geq 1$ for which $\|k \cdot \log_2 3\|$ is smaller than for every smaller k. The closure depths are the denominators of the convergents of $\log_2 3 = [1; 1, 1, 2, 2, 3, 1, 5, 2, 23, 2, 2, 1, 1, 55, \ldots]$: 1, 2, 5, 12, 41, 53, 306, 665, 15601, 31867, 79335, 111202, 190537 up to 10^6.

Proof. The best approximations of the second kind are exactly the convergents (Lagrange; Khinchin, Theorems 16–17). ∎

> F-T18 (THE COMMA) Lean exact
> At depth 12 the two ladders miss by $3^{12}/2^{19} = 531441/524288 = 1.013643$, the Pythagorean comma (23.46 cents). The miss never vanishes (F-T16).

6. The first knot and coupling (F-T19 to F-T24)

These results add stated hypotheses to the axioms.

> F-T19 (FIRST KNOT) exact, partly
> *Hypotheses:* the strand is embedded in three-dimensional space, as a closed curve or with fixed endpoints; crossings are countable; a crossing has three states (over +1, under −1, none 0). The trefoil is the simplest configuration that cannot be undone, at three crossings.

Proof idea. Fox three-colouring (at each crossing one colour or all three: the window rule of F-T4) is invariant under the Reidemeister moves; the trefoil admits 9 colourings, the unknot 3. No knot exists below three crossings. ∎

> F-T20 (BRACKET OF THE FIRST KNOT) exact, partly
> For the trefoil the Kauffman bracket is $\langle K \rangle = A^{-7} - A^{-3} - A^5$ and the Jones polynomial is $V(t) = t + t^3 - t^4$; the mirror gives $t^{-1} + t^{-3} - t^{-4}$. The first knot is chiral.

F-T21 (DOUBLE COVER) exact, partly

Hypothesis: orientation states are unit quaternions and the strand is attached. A strand returns to its initial state after a rotation of 4π, not 2π.

F-T22 (CHARGE CONSERVATION) not verified

Hypotheses: field configurations are maps $S^3 \to S^2$; the strand is continuous; no boundary is crossed. The Hopf charge $Q \in \pi_3(S^2) = \mathbb{Z}$ is conserved under all continuous rewrites, without Noether's theorem.

F-T23 (CONSTITUTIVE EQUATION) exact

Hypothesis: the energy density is $e(n) = (g_2/2)n^2 + (g_3/6)n^3$ with $g_3 \neq 0$, and $\bar{g} = g_2 + g_3\bar{n} \neq 0$. Then $g_{\text{eff}} = \bar{g}(1 + \gamma\delta)$ with $\gamma = g_3\bar{n}/\bar{g}$ and $\delta = (n - \bar{n})/\bar{n}$; and $g_2 = \bar{g}(1 - \gamma)$. By N4, γ is a state function; no fixed value is asserted.

F-T24 (LEVER OF A FREE PATH) exact

Hypothesis: the strand is a simple random walk on $\mathbb{Z}^d$, $d \geq 3$, with return probability $P_d < 1$. The visit count is geometric and its lever is $\gamma = -(1 + P_d)/P_d$: −3.9365, −6.1759 and −8.3976 for $d = 3, 4, 5$.

Open (Foundations). IX.1 whether the lever of a free path equals the coupling γ of the medium; IX.2 the closure dynamics of the whole net; IX.3 the limit of the share in $\mathbb{Z}[\omega]$; IX.4 when blocks have equal size; IX.5 the physical reading of the trit rule.

7. The connection to physics

The *Physics* document uses the axioms and the Foundations theorems. Each statement carries its status. Nothing in this section is needed for Sections 3–6.

7.1 Theorems under a named physical input (conditionally proved)

P-T1 (NILPOTENCE IS THE MASS SHELL) exact

Input: Dirac's equation in Rowlands' nilpotent form, $Q = ikE + i\mathfrak{p} + jm$. Then $Q^2 = (E^2 - p^2 - m^2)\cdot 1$, so $Q^2 = 0$ exactly on the mass shell.

P-T2 to P-T4 (QUATERNION IDENTITIES) exact

$|q + a|^2 - |q - a|^2 = 4\cos(q, a)$ for unit vectors; triple products of pure quaternions reduce to dot products and a determinant; the ring measure $R_Q = \lambda_1((1/k)\Sigma D_i D_i^T)$ of unit quaternions lies in $[¼, 1]$ and is invariant under $D_i \to -D_i$.

> P-T5 (THE BRIDGE) `exact`
> *Input:* Larson's reading of space and time as two aspects of one motion. The map $\phi(x, y, z) = (x - 3z, y + 3z)$ sends the SI dimension $L^xT^yM^z$ to $S^{x-3z}T^{y+3z}$: speed S/T, energy T/S, momentum T^2/S^2, mass T^3/S^3; its kernel is generated by $M{\cdot}L^3/T^3$; Planck's constant has dimension T^2/S.

> P-T6 (LOSS HAS AN ADDRESS) `not verified`
> *Input:* local equilibrium, balance equations, and the second law for every sub-volume. Entropy production $\sigma = \Sigma\ J{\cdot}X \geq 0$ per location.

7.2 Correspondences (declared, not proved; can fail)

- **C1** Spencer-Brown's mark of distinction is the return of the strand to itself.
- **C2** The local state is a quaternion rotor; spin-½ is the rotation label of F-T21.
- **C3** Rowlands' nilpotent condition corresponds to closure.
- **C4** Stable particles are closed windings; the electron corresponds to the Williamson–van der Mark double-loop configuration.
- **C5** The coupling γ is a state function (N4); one geometric turn per mesh. **C5a** $\delta = k\chi_a$ near the reference point.
- **C6** The trit values of a physical site are winding, rest and counterwinding.

7.3 Identifications

- **I1** The electron is the Williamson–van der Mark double-loop photon.
- **I2** The state variable $\chi_a = a/(cH_0)$.
- **I3** $a_0 = cH_0/2\pi = 1.08 \times 10^{-10}$ m s^{-2} for $H_0 = 70$ km s^{-1} Mpc^{-1}; the measured threshold from rotation curves is 1.2×10^{-10} m s^{-2}. The numerical closeness is a known coincidence (Milgrom); the theory's contribution is the reading of 2π as one turn per mesh. I3 becomes a test only when the theory fixes H_0 or the turn independently (O8).
- **I4** $g = g(\alpha(\chi))$: context sets α, α sets interaction.

7.4 Predictions (with pre-commitments)

- **P1** $\Delta\alpha/\alpha \approx +3 \times 10^{-8}$ at the white dwarf G191-B2B, conditional on the pair (χ_Φ, β). Present spectroscopy reaches about 2×10^{-5}, so P1 cannot be decided with present data.
- **P2** Overdetermination: $\gamma k = \beta{\cdot}p_\alpha$ must agree between rotation curves and α-spectroscopy.
- **P3** The solar winding edge at $r = \sqrt{(GM/a_0)} = 7{,}400$ AU (7,030 AU with the measured threshold).
- **P4** No context-independent dark-matter particle will be detected (operational form).
- **P5** Forbidden outcomes: a quantized spin-2 graviton; a residual dipole in α after the χ-regression.

No prediction is confirmed at present.

7.5 Open calculations

O1 coarse-graining of the knot network; O3 the mass spectrum including $m_p/m_e = 1836$, and the electric charge; O4 one state variable χ; O5 Maxwell's equations from N1–N5; O6 Dirac's equation from N1–N5 without input; O7 the fine-structure constant; O8 the Hubble constant or the turn, fixed independently. The reduction of physics to N1–N5 remains a working conjecture.

8. Independent verification

The results of Sections 3–7 were checked independently of the edition, at `maze.swarp.nl/bewezen`. The rule was strict: a result counts only for the part that Lean proves or an exact computation checks. The literal text of the edition is kept beside each result, and everything in it that is not checked is listed as removed. A result counts only if everything its proof depends on also counts. Four rounds of independent review (separate model agents that rebuilt the proofs and recomputed every number) led to the present state. The final round found no claim that went beyond its evidence.

Result	Checked	Not checked (removed)
F-T1–F-T5, F-T10, F-T11	Lean, full statement	—
F-T7, F-T7.1	Lean for every word length; shortest closures exact	—
F-T8, F-T9	Lean for every length; counts exact for $n \le 9$	counts for $n > 9$
F-T12, F-T12.1–12.3	Lean (existence and uniqueness over all lengths; both directions of the digit rule)	infinite expansions (12.2); the log formula beyond $2{\cdot}10^5$
F-T13, F-T14	exact ($n \le 8$; $\lvert a\rvert, \lvert b\rvert \le 100$)	general statements; Burnside in general
F-T15, F-T16, F-T18	Lean (F-T16 for all integer exponents); comma and table exact	density of $k{\cdot}\log_3 2$
F-T17	exact up to $k = 10^6$	Lagrange's theorem in general
F-T19, F-T20, F-T21	exact: 9 colourings; bracket, Jones polynomial and mirror by state sum; q and $-q$ give one rotation	Reidemeister invariance, minimality, chirality conclusion, $\pi_1(\mathrm{SO}(3))$
F-T22	—	whole statement
F-T23, F-T24	exact (symbolic; P_3, P_4, P_5 recomputed)	geometric distribution of visits; Laplace remark
P-T1–P-T5	exact (P-T1 also with Dirac matrices)	bounds $¼ \le R_Q \le 1$
P-T6	—	whole statement

The Lean development uses no external library, contains no `sorry`, `admit` or `native_decide`, and its axiom report lists only propositional extensionality, choice and quotient soundness. The exact computations pass 31 of 31 checks.

9. What the theory does not claim

- The Foundations make no physical claim.
- Dirac's equation is an input, not a result (O6). Maxwell's equations are translated, not derived (O5).
- $a_0 = cH_0/2\pi$ is an identification, not a derivation; its value moves with H_0.
- The coupling γ is a state function with no fixed value.
- No prediction is confirmed.
- Readings of the trit cube as the Clifford algebra Cl(3) (Russell, Larson, Maxwell) are correspondences. Their arithmetic core holds: the 27 cells give the blades of Cl(3), and the 9 closing cells give 1, the three unit bivectors and ±I. But addition of cells is not the product of the algebra, and two cells per bivector is not the double cover (q and $-q$).

10. Reproducibility

The Lean files (`maze/lean/Foundations/Register.lean`, `Adres.lean`), the exact computations (`maze/nieuw/bewezen/exact.py`), the per-result record with the literal text of the edition (`bewezen.json`) and the four review reports are public at `maze.swarp.nl/bewezen`. The canonical editions are identified by their SHA-256 hashes: *Foundations* 9.2, 89315591…; *Physics* 9.0, 56054a20….

Annotated references

1. Spencer-Brown, G. (1969). *Laws of Form*. London: Allen & Unwin.
   The calculus of distinction behind N1 and C1.
2. Kauffman, L. H. (1987). Self-reference and recursive forms. *Journal of Social and Biological Systems*, 10, 53–72.
   Re-entry and eigenforms; the reading of closure as a fixed point (F-T10).
3. Knuth, D. E. (1997). *The Art of Computer Programming, Vol. 2*, 3rd ed., §4.1. Addison-Wesley.
   Balanced ternary and the truncation property used in F-T12.2.
4. Andrews, G. E. (2007). Euler's "De Partitio Numerorum". *Bulletin of the AMS*, 44(4), 561–573.
   Cited in the edition for Euler's product identity as a precedent of the uniqueness in F-T12.
5. Khinchin, A. Ya. (1964). *Continued Fractions*. University of Chicago Press.
   Lagrange's theorem on best approximations, the general proof of F-T17.
6. Hardy, G. H., & Wright, E. M. (2008). *An Introduction to the Theory of Numbers*, 6th ed. Oxford University Press.
   Kronecker's theorem on dense fractional parts (F-T16).
7. Reidemeister, K. (1927). Elementare Begründung der Knotentheorie. *Abh. Math. Sem. Univ. Hamburg*, 5, 24–32.
   The three moves on which F-T19 rests.

8. Przytycki, J. H. (1998). 3-coloring and other elementary invariants of knots. *Banach Center Publications*, 42, 275–295.
   Fox colourings and their relation to the Jones polynomial (F-T19, F-T20).

9. Kauffman, L. H. (1987). State models and the Jones polynomial. *Topology*, 26(3), 395–407.
   The bracket used in F-T20.

10. Jones, V. F. R. (1985). A polynomial invariant for knots via von Neumann algebras. *Bulletin of the AMS*, 12, 103–111.
    The Jones polynomial.

11. Hopf, H. (1931). Über die Abbildungen der dreidimensionalen Sphäre auf die Kugelfläche. *Mathematische Annalen*, 104, 637–665.
    The Hopf invariant behind F-T22.

12. Pólya, G. (1921). Über eine Aufgabe der Wahrscheinlichkeitsrechnung betreffend die Irrfahrt im Straßennetz. *Mathematische Annalen*, 84, 149–160.
    Recurrence and transience of random walks (F-T24).

13. Montroll, E. W. (1956). Random walks in multidimensional spaces, especially on periodic lattices. *J. SIAM*, 4, 241–260.
    Return probabilities in higher dimensions (F-T24); values as in OEIS A086230, A086232, A086233.

14. Dirac, P. A. M. (1928). The quantum theory of the electron. *Proc. R. Soc. A*, 117, 610–624.
    The input to P-T1.

15. Rowlands, P. (2007). *Zero to Infinity: The Foundations of Physics*. World Scientific.
    The nilpotent form of the Dirac operator (P-T1, C3).

16. Maxwell, J. C. (1873). *A Treatise on Electricity and Magnetism*. Oxford: Clarendon Press.
    The quaternion form of the field equations (Part II of Physics, O5).

17. Larson, D. B. (1959). *The Structure of the Physical Universe*. Portland: North Pacific.
    The space–time reading behind P-T5.

18. Williamson, J. G., & van der Mark, M. B. (1997). Is the electron a photon with toroidal topology? *Annales de la Fondation Louis de Broglie*, 22(2), 133–160.
    The configuration named in C4 and I1.

19. Milgrom, M. (1983). A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis. *Astrophysical Journal*, 270, 365–370.
    The acceleration threshold and its closeness to $cH_0/2\pi$ (I3).

20. de Groot, S. R., & Mazur, P. (1962). *Non-Equilibrium Thermodynamics*. Amsterdam: North-Holland.
    Entropy production as $\Sigma\, J{\cdot}X$ (P-T6).

21. The Lean Community (2026). *Lean 4*, version 4.34. lean-lang.org.
    The proof assistant used for the verification in Section 8.