# The Vacuum.Net Theory: Physics

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Version 9.0. Companion: *The Vacuum.Net Theory: Foundations*, Ninth Edition.

This document connects the Vacuum.Net to physics. It uses the axioms N1–N5 and the theorems of the companion document, cited as "Foundations T*n*". Nothing here is needed for the Foundations; everything here depends on them. Each statement carries its status: conditionally proved with an external input named in its hypothesis, corresponded, identified, predicted, or open. Theorems in this document are numbered T1–T6.

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## Part 0 — The Chain Continued

The Foundations end with the first knot and the coupling of the medium. The chain continues in four steps.

1. **Motion.** Dirac's equation in Rowlands' nilpotent form: $Q^2 = 0$ exactly on the mass shell $E^2 = p^2 + m^2$ (T1). Nilpotence is read as closure (C3).
2. **Field.** Maxwell's quaternions carry two registers in one product; the net reads them as two readings of one closure operation (Part II, correspondence; T2–T4 conditionally proved).
3. **Measure.** Space and time are the two aspects of one motion; energy T/S, mass $T^3/S^3$, $h = T^2/S$ (T5). Conversion units are fixed by hydrogen.
4. **Numbers.** $a_0 = cH_0/2\pi = 1.08 \times 10^{-10}$ m s$^{-2}$ for $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$ (I3); the solar winding edge at 7,400 AU (P3); $\Delta\alpha/\alpha \approx 3 \times 10^{-8}$ at G191-B2B (P1).

Steps 1–3 connect the strand to known physics, partly by proof under named inputs and partly by correspondence. Step 4 gives numbers that instruments can check, with their present state of testing.

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## Part I — Nilpotent Rewriting: from Dirac to Closure

### I.1 Input: Dirac's equation

Dirac (1928): $(\gamma^\mu p_\mu - m)\psi = 0$, in natural units with metric (+, −, −, −). Multiplying by $(\gamma^\nu p_\nu + m)$ gives $(E^2 - p^2 - m^2)\psi = 0$, because $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu}$. Dirac's equation enters this document as given input.

## I.2 Rowlands' rewriting

Rowlands replaces the four gamma matrices by two commuting sets of units. The quaternion units **i**, **j**, **k** square to −1 and anticommute. The vector units square to +1 and anticommute; they are carried by the momentum vector **p**. Each unit of one set commutes with each unit of the other. A third ingredient is the scalar imaginary unit i ($i^2 = -1$), which commutes with all six units; it is the i in i**k**E and in the operator, while the **i** of **ip** is the quaternion unit. The Dirac operator becomes the nilpotent

$$Q = i\mathbf{k}E + \mathbf{ip} + \mathbf{j}m,$$

and the equation reads $(\mp\mathbf{k}\partial_t \mp i\mathbf{i}\nabla + \mathbf{j}m)(\pm i\mathbf{k}E \pm \mathbf{ip} + \mathbf{j}m)e^{-i(Et - p\cdot r)} = 0$.

**T1 (Nilpotence is the mass shell).** $Q^2 = (E^2 - p^2 - m^2) \cdot 1$. Hence $Q^2 = 0$ exactly when $E^2 = p^2 + m^2$. Status: conditionally proved (hypothesis: Dirac's equation in Rowlands' form); computed in a 4 × 4 representation, on and off the shell.

*Proof.* $(i\mathbf{k}E)^2 = -\mathbf{k}^2E^2 = E^2$. $(\mathbf{ip})^2 = \mathbf{i}^2\mathbf{p}^2 = -p^2$, because the vector units square to +1 and anticommute, so $\mathbf{p}^2 = p^2$. $(\mathbf{j}m)^2 = -m^2$. There are three cross terms, each an anticommutator:

- i**k**E with **ip**: $iE(\mathbf{ki} + \mathbf{ik})\mathbf{p} = 0$, since $\mathbf{ki} = -\mathbf{ik}$;
- i**k**E with **j**m: $iEm(\mathbf{kj} + \mathbf{jk}) = 0$, since $\mathbf{kj} = -\mathbf{jk}$;
- **ip** with **j**m: $m(\mathbf{ij} + \mathbf{ji})\mathbf{p} = 0$, since $\mathbf{ji} = -\mathbf{ij}$.

In each line **p** and the scalar i commute with **i**, **j**, **k** and can be moved out; what remains is an anticommutator of two different quaternion units, which is zero. So $Q^2 = E^2 - p^2 - m^2$. ∎

Computed: in the 4 × 4 representation (quaternion units $-i\sigma \otimes 1$, vector units $1 \otimes \sigma$) each of the three anticommutators is the zero matrix, and $Q^2 = (E^2 - p^2 - m^2) \cdot 1$ to $10^{-14}$. An independent check with Dirac matrices ($Q = \gamma^0 E + \boldsymbol{\gamma}\cdot\mathbf{p} + i\gamma^5 m$) gives the same identity to $4 \times 10^{-15}$.

## I.3 The link to the net

The three units of Q are three floors E, p, m with signature (+, −, −). Closure applied to itself adds nothing: $\Psi^2 = 0$. C3 identifies this closure with Rowlands' nilpotence. The chain is: Dirac (input) → nilpotent form (Rowlands) → $Q^2 = 0$ ↔ closure (C3) → $E^2 = p^2 + m^2$ (T1).

Status. T1 is conditionally proved. The identification of nilpotence with closure is a correspondence (C3). A derivation of Dirac's equation from N1–N5 without input is open (O6).

# Part II — Maxwell's Quaternions: Two Registers

## II.1 The record

Hamilton's operator $\nabla = \mathbf{i}\partial_x + \mathbf{j}\partial_y + \mathbf{k}\partial_z$ acts on a vector with one product. The product has two parts: a scalar part, the convergence, and a vector part, the curl. Maxwell wrote his grouped equations of 1873 (*Treatise*, Arts. 591–619) in this form: $\mathfrak{B} = V\nabla\mathfrak{A}$ (B = ∇ × A); $e = S\nabla\mathfrak{D}$ (J) as printed, read today as $\rho = \nabla\cdot D$ (with Hamilton's sign convention $S\nabla\mathfrak{D} = -\nabla\cdot D$, the convergence; the printed form carries Maxwell's sign); $\mathfrak{C} = \mathfrak{K} + \mathfrak{D}$ (H), the total current with the displacement term (in Maxwell $\mathfrak{J}$ denotes the intensity of magnetization); and the constitutive moduli K, C, μ, with $c^2 = 1/(K\mu)$ (Art. 784 ff.). Gibbs (*Elements of Vector Analysis*, 1881–84) and Heaviside (1882–85) split the one product into dot and cross; Heaviside's four-equation vector form dates from 1884–85. The scalar register disappeared. In this document's reading, K, C and μ were then treated as fixed constants; in Maxwell (Art. 618) they are coefficients of the medium.

## II.2 The rewriting in net terms (correspondence)

| Maxwell | Net |
|---|---|
| one product, two readings (S∇ and V∇) | two readings of one closure operation: the two-layer law |
| potential $\mathfrak{A}$ first, fields as readings of it | the winding state of the mesh is the object; fields are readings |
| free charge $e = S\nabla\mathfrak{D}$ | the scalar residue of non-closure, written as a trit +1/0/−1 |
| displacement term | the closure-completing term: the total current always closes |
| moduli K, C, μ | state functions g(χ) of the local vacuum (N4) |
| $c^2 = 1/(K\mu)$ | c as a state function, not a postulate |
| non-commutative product | order is orientation (N3); a full return takes 4π (Foundations T21) |

Status: correspondence. Maxwell's equations are translated into the net, not derived from N1–N5 (O5).

## II.3 The composition rule (conditionally proved)

A passage question → neighbour → answer is read with quaternions. Two lemmas show why the scalar register is needed.

**T2 (The 4c identity).** For unit vectors q, a with c = cos(q, a): $|q + a|^2 - |q - a|^2 = 4c$. Status: conditionally proved.

*Proof.* $|q \pm a|^2 = 2 \pm 2c$. ∎

**T3 (Pure quaternions collapse).** With pure quaternions Q = (0, q), N = (0, n), A = (0, a) and |q| = 1, every term of the triple product reduces to pair dot products (cosines when n and a are unit) or to det[q, n, a]: S(QNA) = −det[q, n, a]; V(QNA) = −(q·n)a + (q·a)n − (n·a)q; −V(QNA)·q = n·a; S(QNAQ) = n·a. Status: conditionally proved.

*Proof.* QN = (−q·n, q × n). Expand with (u × v) × w = v(u·w) − u(v·w) and the Binet–Cauchy identity. ∎

So a rule that keeps orientation needs full quaternions, scalar part included, and one frame per neighbour. With frames $e_1^{(i)}$, $e_2^{(i)}$, $e_3^{(i)}$ per sister cell, each item becomes $Q^{(i)} = (\cos(\theta/2), \sin(\theta/2) \cdot w)$, the shortest rotation that carries the region centre of cell i to q (angle θ, unit axis w), with $A^{(i)}$ likewise for a. The relative winding is $D_i = (Q^{(i)})^* \cdot A^{(i)}$.

**T4 (Ring measure).** For unit quaternions $D_1, \ldots, D_k$, put $B = (1/k) \Sigma D_i D_i^T$ and $R_Q = \lambda_1(B)$. Then ¼ ≤ R_Q ≤ 1, and R_Q is unchanged when any $D_i$ is replaced by $-D_i$. Status: conditionally proved.

*Proof.* Each $D_i D_i^T$ has trace 1, so trace B = 1 and B is positive semidefinite. Its largest of four eigenvalues lies in [¼, 1]. $(-D)(-D)^T = DD^T$. ∎

The sign invariance is the double cover (Foundations T21) at work: q and −q give the same rotation. Worked example: q = (1, 0, 0), a = (0, 1, 0), region centres (1, 1, 0)/√2 and (1, 0, 1)/√2, 60° apart. For the first cell, $Q^{(1)}$ and $A^{(1)}$ are rotations by 45° about ∓z, which gives $D_1$ = (0.707, 0, 0, 0.707). For the second, $Q^{(2)}$ = (0.924, 0, 0.383, 0) and $A^{(2)}$ = (0.707, −0.5, 0, 0.5), which gives $D_2$ = (0.653, −0.653, −0.271, 0.271). Then $D_1 \cdot D_2$ = 0.653 and R_Q = (1 + 0.653)/2 = 0.827. Computed.

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# Part III — Units and Dimensions

The model is dimensionless. Its native quantities are ratios: winding sums, densities relative to reference, χ_a. There is no ħ in the model. c is a state function of the medium, not a universal input. SI numbers are conversions performed after the fact, for comparison with instruments. By Foundations T12.2, finite-depth readout is self-centering: the notation needs no external rounding convention. Because its quantities are dimensionless ratios, it needs no unit convention either.

## III.1 The dimensional ladder (Larson)

Conversions follow Larson’s reading of motion: space S and time T as the two aspects of one motion. Motion is S/T. Energy is T/S. Mass is $T^3/S^3$. Momentum is $T^2/S^2$.

**T5 (The bridge).** The map φ(x, y, z) = (x − 3z, y + 3z) sends the SI dimension $L^x \cdot T^y \cdot M^z$ to $S^{x-3z} \cdot T^{y+3z}$. It sends speed to S/T, energy to T/S, momentum to $T^2/S^2$ and mass to $T^3/S^3$. Energy per unit time is 1/S. Mass times motion squared is energy. The exchange of S and T is an involution that maps motion to energy. The kernel of φ is generated by $M \cdot L^3/T^3$. Status: conditionally proved.

*Proof.* φ(1, −1, 0) = (1, −1); φ(2, −2, 1) = (−1, 1); φ(1, −1, 1) = (−2, 2); φ(0, 0, 1) = (−3, 3). (T/S)/T = 1/S. (T³/S³)(S/T)² = T/S. x − 3z = 0 and y + 3z = 0 give (x, y, z) = z(3, −3, 1). ∎

Planck's constant. E = hν with E = T/S and ν = 1/T gives h = T²/S.

### III.2 The conversion units

For comparison with instruments the conversions use one length and one time, both fixed by hydrogen: $S_0 = 1/(2R_H)$, with $R_H = R_\infty/(1 + m_e/m_p)$, and $T_0 = S_0/c$. Inputs (CODATA 2018): $R_\infty$ = 10,973,731.568160 m⁻¹, $m_p/m_e$ = 1,836.15267343, c = 299,792,458 m/s, h = 6.62607015 × 10⁻³⁴ J·s.

| Unit | Definition | Value |
|---|---|---|
| $R_H$ | $R_\infty/(1 + m_e/m_p)$ | 10,967,758.340 m⁻¹ |
| $S_0$ | $1/(2R_H)$ | 4.55881671 × 10⁻⁸ m |
| $T_0$ | $S_0/c$ | 1.52065757 × 10⁻¹⁶ s |
| $E_0$ | $h/T_0 = 2hcR_H$ | 4.35737162 × 10⁻¹⁸ J = 27.1965745 eV |

$E_0$ is the Hartree energy of hydrogen with its reduced mass. On the ring ladder (Foundations T15 with a = 0) the layer at depth n has length $3^n \cdot S_0$ and time $3^n \cdot T_0$; on a mixed ladder the factor is $2^a \cdot 3^b$. At ring depth 2 the length is $9S_0$ = 410.29 nm, the vacuum wavelength of the Balmer Hδ line of hydrogen with its reduced mass. That identity holds by construction of $S_0$: the line n = 6 → 2 has $1/\lambda = R_H(1/4 - 1/36) = 2R_H/9$. It is a consistency of the unit, not an independent agreement.

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## Part IV — Loss Has an Address

**T6 (Loss has an address).** *Hypotheses.* Local equilibrium holds per mesh (Gibbs relation); local balance equations hold; the second law holds for every sub-volume; fluxes J and forces X are defined per mesh; channels may couple. *Statement.* Entropy production is $\sigma = \Sigma\, J \cdot X \geq 0$ for the total, per location. In the linear regime J = LX with Onsager-symmetric L, the individual terms are sign-definite in the channels that diagonalize L; in coupled channels a single $J_i X_i$ can be negative. σ = 0 does not force every X to zero: in the linear regime it forces J = LX = 0, with X free in the kernel of L; outside the linear regime it constrains only the total. Status: conditionally proved.

*Proof sketch.* Under local equilibrium, the Gibbs relation and the balance equations give $\sigma = \Sigma\, J \cdot X$ (de Groot–Mazur). The second law applied to every sub-volume, however small, gives σ ≥ 0 pointwise. Positivity binds the sum, not the parts. For J = LX with symmetric L, $\sigma = X^T L X$. Since σ ≥ 0 for every X, L is positive semidefinite. In an orthonormal eigenbasis of L, $J'_k = \lambda_k X'_k$, the pairing is preserved ($\Sigma\, J'_k X'_k = \Sigma\, J_i X_i$), and $\sigma = \Sigma\, \lambda_k X'^2_k$ with $\lambda_k \geq 0$; each term $J'_k X'_k = \lambda_k X'^2_k \geq 0$. ∎

Consequence: every irreversible discharge occurs at a mesh with an address. Loss is bookkeeping on the net, not a leak out of it.

## Part V — Correspondences

Correspondences identify model elements with observed structures. They are neither proved nor predicted; they are declared, and they can fail.

**C1 (Mark).** Spencer-Brown's mark of distinction = the return of the strand to itself. Absorbed into N1.

**C2 (Rotor).** The local state is a quaternion rotor; spin-½ is the rotation label of Foundations T21.

**C3 (Nilpotence).** Rowlands' nilpotent condition $Q^2\psi = 0$ with $Q^2 = E^2 - p^2 - m^2$ corresponds to closure: a configuration closes exactly when its rotor squares to zero in the Cl(1,2) fragment.

**C4 (Knot = matter).** Stable particles are closed windings. The electron corresponds to the Williamson–van der Mark toroidal configuration: a photon closed on itself in a double loop.

**C5 (Coupling and crossover).** Two roles are separated. (i) The coupling exponent γ of Foundations T23 is a state function (N4), not a fixed number. For a free path Foundations T24 gives the lever $\gamma = -(1 + P_d)/P_d$; whether this lever is the γ of the medium is open (O1). (ii) The crossover keeps one geometric turn per mesh: one mesh around is one full turn, 2π. The crossover rule $\chi_a = 1/2\pi$ gives $a_0 = cH_0/2\pi$ (I3).

**C5a (Bridge, observable–state).** $\delta = k\chi_a$ near the reference point, with $k \equiv d\delta/d\chi_a$. k = 1 is an additional choice, not a result. The measured threshold tests I3; it does not fix γ or k separately. Validity: near the reference point only.

**C6 (Trit = physical winding).** In the Foundations the trit is defined from σ by the trit rule. What remains a correspondence is that the trit values {1, 0, T} of a physical site are {winding, rest, counterwinding}: that physical windings are passages of the strand in the sense of the Foundations.

## Part VI — Identifications

**I1.** Electron = Williamson–van der Mark double-loop photon.

**I2.** State variable $\chi_a = a/(cH_0)$: the only independent dimensionless ratio of local acceleration to background.

**I3.** $a_0 = cH_0/2\pi$, with 2π the geometric turn of C5 (ii). With $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$ = $2.2685 \times 10^{-18}$ s$^{-1}$: $a_0 = 1.0824 \times 10^{-10}$ m s$^{-2}$, written $1.08 \times 10^{-10}$ m s$^{-2}$. Measured threshold (rotation-curve fits): $1.2 \times 10^{-10}$ m s$^{-2}$. The value moves with the Hubble constant:

| $H_0$ (km s$^{-1}$ Mpc$^{-1}$) | Source | $a_0 = cH_0/2\pi$ (m s$^{-2}$) | Below $1.2 \times 10^{-10}$ by |
|---|---|---|---|
| 67.4 | Planck 2018 | $1.04 \times 10^{-10}$ | 13 % |

| $H_0$ (km s$^{-1}$ Mpc$^{-1}$) | Source | $a_0 = cH_0/2\pi$ (m s$^{-2}$) | Below $1.2 \times 10^{-10}$ by |
|---|---|---|---|
| 70.0 | reference value | $1.08 \times 10^{-10}$ | 10 % |
| 73.0 | distance ladder (SH0ES) | $1.13 \times 10^{-10}$ | 6 % |

The numerical closeness of $a_0$ to $cH_0/2\pi$ is a known coincidence in the MOND literature (Milgrom 1983, 1999). The contribution of this theory is the reading of the 2π as one turn per mesh (C5 ii), not the coincidence itself. I3 becomes a test only when the theory fixes $H_0$ or the turn independently.

**I4.** $g = g(\alpha(\chi))$: context sets α, α sets interaction. $\Delta\alpha/\alpha = \beta \cdot \Delta\chi$ is the inner function in the weak-field tail.

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## Part VII — Predictions

Each prediction carries its pre-commitments. A prediction without a pre-registered map is not a prediction.

**P1 (Clock–spectroscopy benchmark, conditional).** In potential form: $\Delta\alpha/\alpha \approx +3 \times 10^{-8}$ at the white dwarf G191-B2B, conditional on the pair ($\chi_\Phi$, β). Inputs: $\Delta\chi_\Phi = GM/(Rc^2)$ at the surface, with $M = 1.0 \times 10^{30}$ kg (0.50 M⊙) and $R = 1.4 \times 10^7$ m: $6.674 \times 10^{-11} \times 1.0 \times 10^{30} / (1.4 \times 10^7 \times 8.988 \times 10^{16}) = 5.30 \times 10^{-5}$; and $\beta = \alpha/4\pi = 0.0072974/12.566 = 5.807 \times 10^{-4}$. By I4, $\Delta\alpha/\alpha = \beta \cdot \Delta\chi_\Phi = 5.807 \times 10^{-4} \times 5.30 \times 10^{-5} = 3.08 \times 10^{-8}$. The published stellar parameters (0.52 M⊙, 0.0204 R⊙; Hu et al. 2021) give $\Delta\chi_\Phi = 5.41 \times 10^{-5}$ and $\Delta\alpha/\alpha = 3.14 \times 10^{-8}$; the prediction is $\approx 3 \times 10^{-8}$ either way. A null result refutes that pair, not the theory. Pre-commitment: the χ-map of the line of sight, fixed before analysis.

*Present state of testing.* The most detailed analysis of G191-B2B (Hu et al. 2021, Fe V lines) gives $\Delta\alpha/\alpha = (6.36 \pm 0.33_{\text{stat}} \pm 1.94_{\text{sys}}) \times 10^{-5}$ with one set of laboratory wavelengths and $(4.21 \pm 0.47_{\text{stat}} \pm 2.35_{\text{sys}}) \times 10^{-5}$ with another. The uncertainty is about $2 \times 10^{-5}$, dominated by the laboratory wavelengths. The predicted value lies about three orders of magnitude below that precision. P1 cannot be decided with present data; it requires laboratory wavelengths roughly a thousand times more precise.

**P2 (Overdetermination).** $\gamma k = \beta \cdot p_\alpha$, with $p_\alpha \equiv d \ln g / d \ln \alpha$. Two independent routes — rotation curves (SPARC) and α-spectroscopy — must return the same product γk at the same reference state (which requires one χ, O4). Sign conditions stated; preregistration required.

**P3 (Solar winding edge).** The edge lies where internal acceleration meets the background: $GM/r^2 = a_0$, hence $r_{edge} = \sqrt{(GM/a_0)}$. Inputs: $GM_\odot = 1.327 \times 10^{20}$ m$^3$ s$^{-2}$ and $a_0 = 1.0824 \times 10^{-10}$ m s$^{-2}$ (I3, $H_0 = 70$). Then $r_{edge} = \sqrt{(1.226 \times 10^{30})}$ m $= 1.107 \times 10^{15}$ m = 7,400 AU. Scaling: $r_{edge} \propto \sqrt{M}$ and $\propto 1/\sqrt{a_0}$; with the measured threshold

$1.2 \times 10^{-10}$ m s$^{-2}$ the edge lies at 7,030 AU. Honest label: the scaling follows from the threshold identification; the theory's own contribution is the value of the threshold (I3).

**P4 (No dark-matter particle).** Operational form: no context-independent particle will ever be detected. Pre-registered function: $\Delta m_{eff}/m_{eff} = s_m \cdot \gamma k \cdot \Delta\chi_a$, with susceptibility $s_m \equiv d \ln m_{eff} / d \ln g_{eff}$, to be computed in O1. A collective excitation that tracks $\chi$ does not count as a particle. Only refutable in this operational form.

**P5 (Forbidden outcomes).** (i) A quantized spin-2 graviton. (ii) Any measured $\alpha$-variation must survive the $\chi$-regression; a residual dipole after regression refutes the model.

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## Part VIII — Open Calculations

**O1. Coarse-graining of the knot network.** $\gamma$ is a state function, and for a free path Foundations T24 gives its value. Open: that the lever of the path is the coupling of the medium; the sign of $g_2 = \bar{g}(1 - \gamma)$ at the reference state, negative for $\gamma > 1$ and positive for $\gamma < 1$ (as for the free-path lever); and P4, which waits on $s_m$.

**O3. The mass spectrum** from the winding alphabet, including $m_p/m_e = 1836$, and the electric charge.

**O4. One $\chi$.** Unify $\chi_a = a/(cH_0)$ and $\chi_\Phi = |\Phi|/c^2$, including k (C5a).

**O5. Maxwell's equations from N1–N5**, beyond the translation of Part II.

**O6. Dirac's equation from N1–N5** without input (Part I).

**O7. The fine-structure constant $\alpha$**, now an input (P1).

**O8. The Hubble constant or the turn**, fixed independently, so that I3 becomes a test rather than a coincidence.

(O2, closure dynamics, is a question of the Foundations, IX.2.)

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## Part IX — Epistemic Statuses

- **Assumed** — the axioms N1–N5 of the Foundations, and the external inputs named in a hypothesis (Dirac's equation in T1; the energy density and thermodynamic hypotheses in T6).
- **Conditionally proved** — T1–T6. T2–T4 are identities of quaternion algebra; T5 is a linear map whose reading as Larson's dimensions is stated in III.1.
- **Corresponded** — C1–C6, declared identifications that can fail. I1–I4 are identifications of the same kind.
- **Predicted** — P1–P5, with pre-commitments and, for P1, the present state of testing.
- **Open** — O1, O3–O8; the reduction of physics to N1–N5 remains a working conjecture.

## Appendix — Record of the Checks

Every theorem and every number in this document was checked with code written for the purpose. The row marked † was checked on 17 October 2026 for the Eighth Edition and is taken over without re-running; all other rows were run on 27 October 2026.

| Item | What was computed | Range or method | Result |
|---|---|---|---|
| T1 | $Q^2$ and its three cross terms | $4 \times 4$ matrices (quaternions ⊗ Pauli; and Dirac matrices); 1000 random points on and off the shell | agrees to $10^{-14}$ |
| T2–T3 | 4c identity; pure-quaternion identities | symbolic; $10^3$ random triples | agrees to $2 \times 10^{-15}$ |
| T4 | ring measure; worked example | construction $D = Q^*{\cdot}A$ from the region centres | $D_1$, $D_2$ as printed; $R_Q = 0.827$ |
| T5 | bridge map | Smith normal form; symbolic dimensions | agrees; kernel $z(3, -3, 1)$ |
| III.2 | conversion units; $9S_0$ | 40-digit decimals | values of the table; 410.29 nm |
| T6 † | entropy production | $10^4$ random positive semidefinite L | agrees |
| I3 | $a_0$ for $H_0 = 67.4$, 70, 73 | stated inputs | 1.04, 1.08, $1.13 \times 10^{-10}$ m s$^{-2}$ |
| P1 | $\Delta\alpha/\alpha$ | both parameter sets | $3.08 \times 10^{-8}$; $3.14 \times 10^{-8}$ |
| P3 | solar edge | $a_0 = 1.0824$ and $1.2 \times 10^{-10}$ | 7,400 AU; 7,030 AU |

## Annotated References


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